Equation of a Line (Functional Form)

Recall : How to calculate the slope of a line given two points

A (x_1, y_1) and B (x_2, y_2) Slope $= \frac{y_2 - y_1}{x_2 - x_1}$

- Positive Slope – increase
- Negative Slope decrease

The equation of a line is functional form:

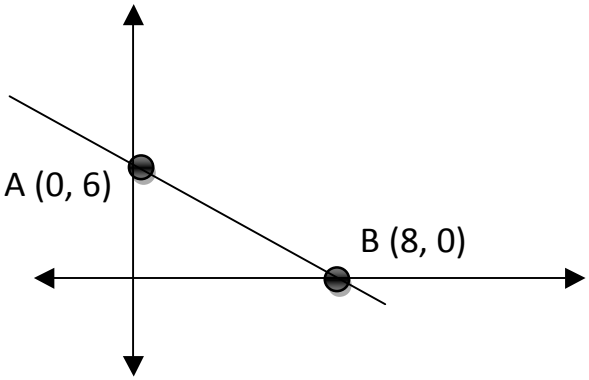
$y = ax + b$

b= y-intercept or initial value

a = slope

Vocabulary:

- The y-intercept is the value on the y-axis where the line crosses the y-axis
- The x-intercept is the value on the x-axis where the line crosses the x-axis



Slope is negative because it is going downwards

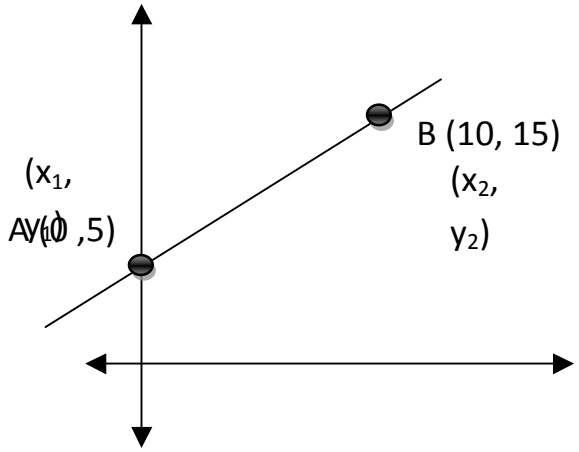
Slope $= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0 - 6}{8 - 0}$

$= \frac{-6}{8}$ or -0.75

From a graph, how do we determine the equation of a line?

1. Determine two points on the line that a given or easy to find (perfect points) and label them.



2. Calculate the slope $a = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{15 - 5}{10 - 0}$

$= \frac{10}{10}$ or 1

3. Calculate the value of the y-intercept $b = y_1 - a(x_1)$

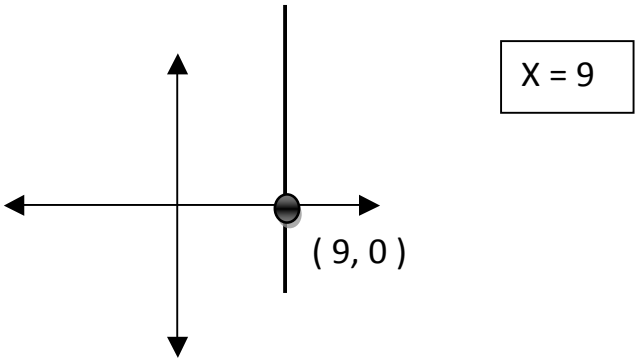
$= 5 - 1(0)$

$= 5 - 0$

b = 5

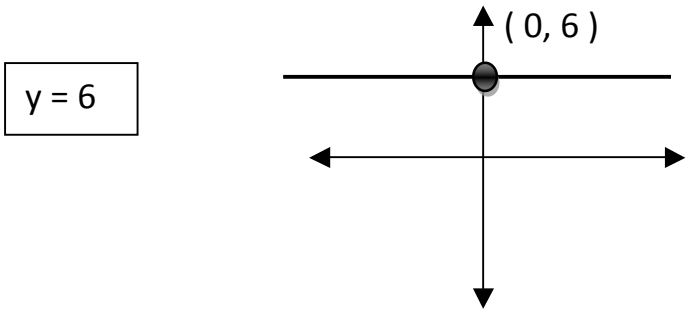
Sometimes we encounter **vertical lines or horizontal lines**

The equation for a **vertical** line is x = c ; where c is the x-intercept



*** the slope of this line is said to be "undefined" or infinite ***

The equation for a **horizontal** line is y = c ; where c is the y-intercept



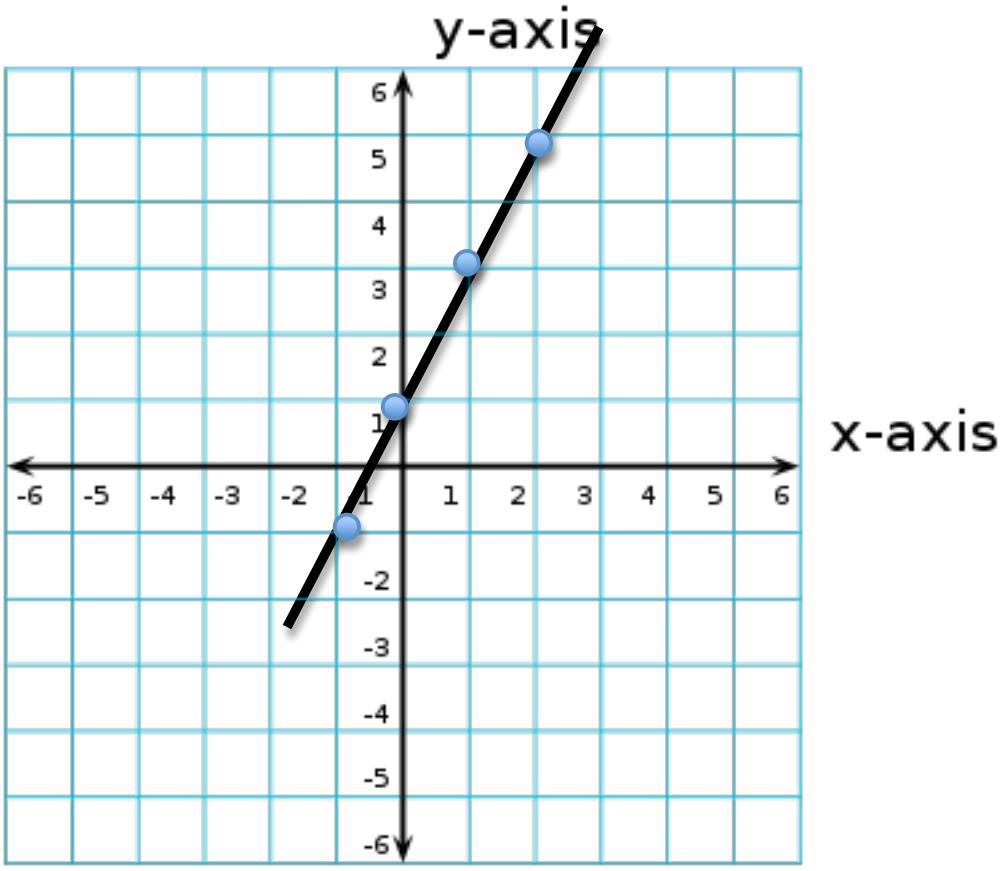
*** the slope of this line is equal to 0 ***

Graphing a Line

Here are a few methods for graphing a line that is given in **functional form**. Choose the way that works for you!

A. **Table of Values** : enter the x-values into the equation to get the y-values. Plot the points and join in a straight line.

Equation: y = 2x + 1



X	Y
-1	-1
0	1
1	3
2	5

B. **Rise over Run (slope!)**

Equation: y = 3x – 5

- Plot the y-intercept

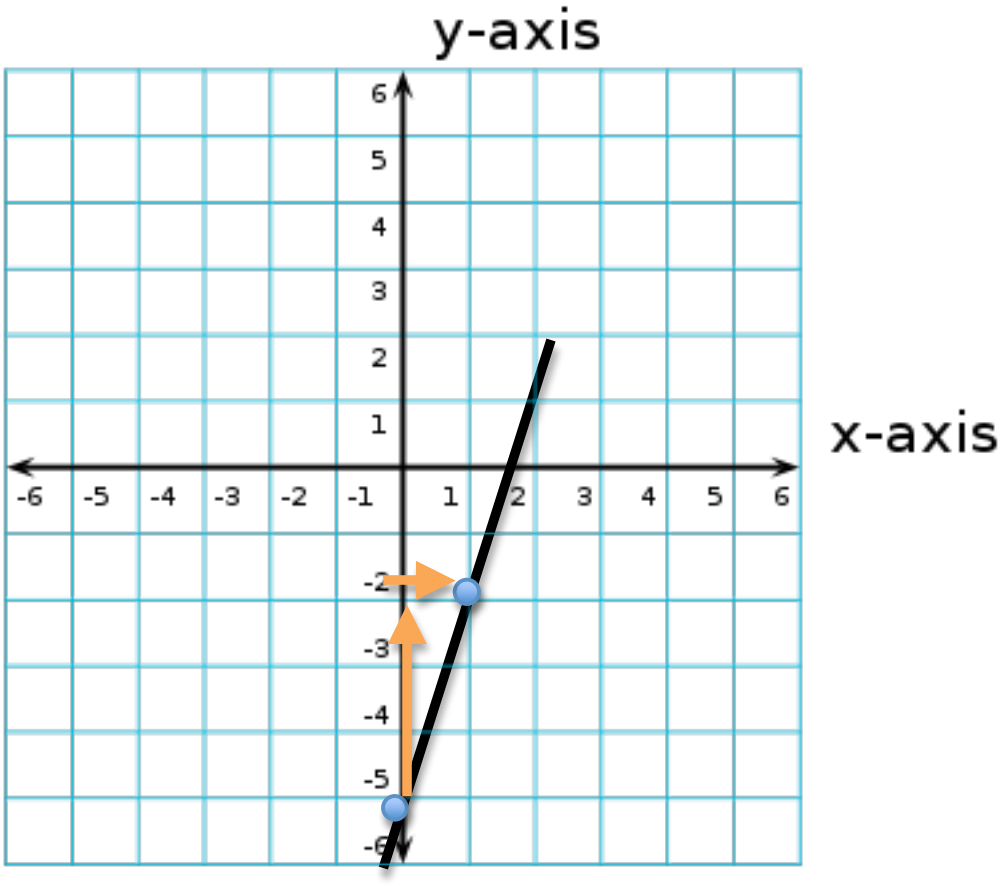
- Starting from the y-intercept…

  move up/down (RISE) by the number of squares on the grid

  then **right/left (RUN)**

3 = 3 (rise = up 3)

1 (run = right 1)



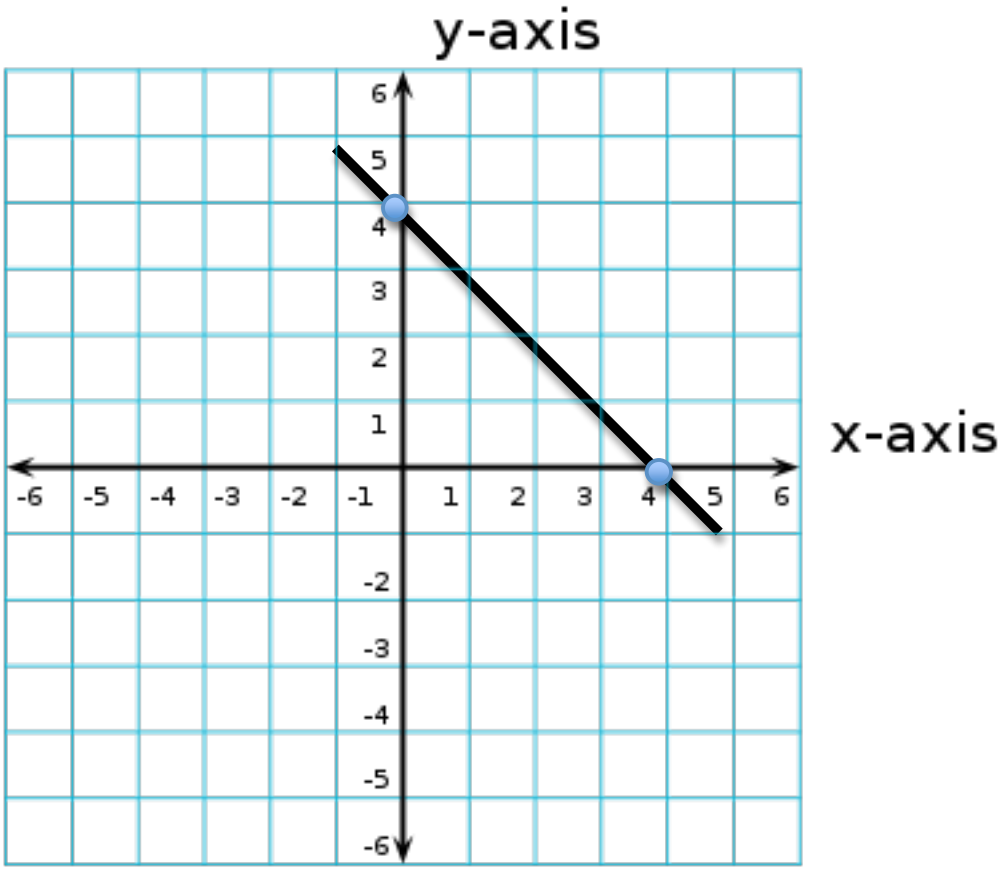
C. **Intercept Method** Equation: y = -x + 4

- Plot the y-intercept (y = 4)
- Determine the x-intercept: replace y with 0 and solve for x!

  0 = -x + 4

  x = 4

- Plot the x-intercept and join in a straight line to the y-intercept



General Form Equation of a Line

Looks like : $ax + by + c = 0$

a is NOT the slope

b is NOT the y-intercept

So, how do we find the slope and the y-intercept?

Example:

$2x + 4y - 6 = 0$

Step 1 – Change the equation to functional form by isolating the y variable.

$2x + 4y - 6 = 0$ ⇒ $4y = -2x + 6$

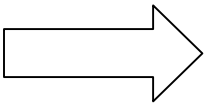
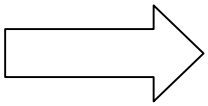
Step 2 – Divide both sides of the equation by the coefficient in front of y to get the equation back into the form of y = ax + b

$$\frac{4y}{4} = \frac{-2x + 6}{4} \Rightarrow y = -0.5x + 1.5$$

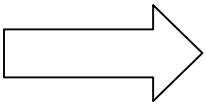
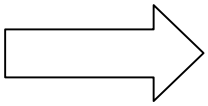
Slope = -0.5 or -1/2

Y-Intercept = 1.5

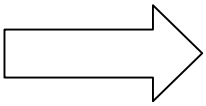
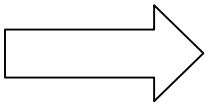
Example #2

$3x - 6y + 15 = 0$  $-6y = -3x - 15$  $Y = \frac{1}{2} x + 2.5$

Example #3

$-30 + 10y = -2x$  $10y = -2x + 30$  $y = -0.2x + 3$

Example #4

$0 = 5y - x$  $-5y = -x$  $y = 0.2x$

Drawing a Line Using the General Form Equation

We use the intercept method, which requires us to find both the x and y intercepts.

If we remember, the x intercept is the point on the line at which y = 0 and the y intercept is the point on the line at which x = 0

Therefore we need to replace each variable with 0 and solve for the other.

Example:

$2x + 6y - 18 = 0$

Make x = 0

$2(0) + 6y - 18 = 0$

Isolate and solve for y

$6y = 18 \Rightarrow \frac{6y}{6} = \frac{18}{6} \Rightarrow y = 3$

This is the y-intercept or point (0,3)

Make y = 0

$2x + 6(0) - 18 = 0$

Isolate and solve for x

$2x = 18 \qquad \frac{2x}{2} = \frac{18}{2} \qquad x = 9$

This is the x-intercept or point (9,0)

With these two points, you can plot them on a Cartesian plane and connect the two points with a line.

Try these examples...

$3x + 6y - 12 = 0$

$2x + y = 6$

$X - 3y + 9 = 0$

$5x - 5y + 5 = 0$

$2y + 4x = 8$

$4y = 16$